🚀 Planetary Position Calculation Guideline

The most precise way to determine a planet's position for any given time t using its **osculating orbital elements** involves several key computational steps. The result will be a position vector r in a heliocentric coordinate system.

1. Calculate Time from Epoch

First, define a reference epoch T_0 (e.g., J2000.0, which is January 1, 2000, 12:00).

Calculate the time difference Δt between the current time t and the epoch T_0. This is often expressed in Julian Days or Julian Centuries (T).

$$\Delta t = t - T_0$$

2. Update Time-Variable Orbital Elements

For higher accuracy, especially over a long period, you must account for the fact that orbital elements are not constant (due to perturbations from other planets). The *osculating orbital elements* are usually provided as a polynomial function of Julian Centuries (T) from the epoch. However, for this project, you can use the constant orbital elements which should produce accurate predictions over a short period of time.

The main orbital elements you'll need are:

- $\boldsymbol{a}$: Semi-major axis
- $\boldsymbol{e}$: Eccentricity
- $\boldsymbol{i}$: Inclination
- $\boldsymbol{\Omega}$: Longitude of the ascending node
- $\boldsymbol{\omega}$: Argument of periapsis
- $\boldsymbol{M}$: Mean anomaly at the epoch

For a planet, a and e may be considered constant for a basic simulation, but the angles (Ω, ω, M) change over time. You'll need the rate of change ($\dot{\Omega}$, $\dot{i}$, etc.) and apply it:

$$Element_t = Element_0 + \dot{Element} \times T$$

3. Solve Kepler's Equation for Position

This is the core of the calculation, involving three key anomalies (angles).

A. Calculate the Mean Anomaly (M)

The mean anomaly represents a fictional angle that increases uniformly with time.

$$M = M_0 + n \times \Delta t$$

Where M_0 is the mean anomaly at the epoch, and n is the mean motion (average angular speed):

$$n = \frac{2\pi}{P}$$

here P is the orbital period.

B. Solve for the Eccentric Anomaly (E)

The eccentric anomaly E is related to M and e by Kepler's Equation:

$$M = E - e \sin E$$

This is a transcendental equation and must be solved iteratively, for example, using the Newton-Raphson method.

C. Calculate the True Anomaly (ν) and Heliocentric Distance (r)

The True Anomaly ν is the actual angle between the planet's position and the perihelion. The heliocentric distance r is the distance from the Sun to the planet.

$$\tan\frac{\nu}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}$$

$$r = a(1 - e\cos E)$$

4. Convert to Heliocentric Cartesian Coordinates

Finally, convert the polar coordinates (r and ν) within the orbital plane to 3D Cartesian coordinates $r = (x, y, z)$ in the reference frame (usually the Heliocentric Ecliptic Coordinate System).

A. Position in Orbital Plane (x', y')

The coordinates (x', y') are defined with the x'-axis pointing to the periapsis.

$$x' = r\cos\nu$$
$$y' = r\sin\nu$$
$$z' = 0$$

B. Rotate to Ecliptic Plane

Rotate this vector by the three Keplerian angles:

1. Argument of Periapsis (ω): Rotation around the z'-axis by ω.
2. Inclination (i): Rotation around the new x'-axis by i.
3. Longitude of the Ascending Node (Ω): Rotation around the new z'-axis by Ω.

The final position $r = (x, y, z)$ is then obtained by applying the rotation matrices (in the order Ω, i, ω to x', y', z'):

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R_3(-\Omega) \cdot R_1(-i) \cdot R_3(-\omega) \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

(Note: The actual formula uses the negative angles on the resulting matrix multiplication, or the angles ω, i, Ω on the inverse transformation, depending on the chosen convention. Look up "Orbital Elements to Cartesian Coordinates" for the correct combined rotation matrix.)

5. Implementing Rotation in Unity

Planetary rotation is simpler, as the period and axial tilt are considered constant for a basic simulation.

1. Define Rotation Axis: The tilt (obliquity, ϵ) defines the angle of the planet's rotational axis relative to the ecliptic pole (perpendicular to the orbital plane).
   - Define the rotation axis vector, typically rotated by the planet's obliquity ϵ from the z-axis of the ecliptic system.
2. Calculate Current Rotation Angle (θ):

$$\theta(t) = \theta_0 + \frac{360}{P_{rot}} \times \Delta t$$

Where P_{rot} is the rotation period in days (or seconds), and θ_0 is the rotation angle at the epoch.

3. Apply Rotation in Unity:
   - Set the planet's object position to the calculated r.
   - Apply the axial tilt as a fixed parent rotation.
   - Apply the current rotation angle $\theta(t)$ around the defined axis (e.g., in a script's *Update()* function).

💾 Suitable Data Sources

For a realistic and accurate model, you will need three types of data:

1. High-Accuracy Orbital Elements / Ephemeris

For the highest accuracy, use full ephemeris data rather than constant orbital elements, as these account for all gravitational perturbations.

- JPL HORIZONS System:
    - Source: NASA Jet Propulsion Laboratory (JPL).
    - Data Type: Provides highly accurate ephemeris (position and velocity vectors) for any object in the solar system for any given time.
    - How to use: The student can use the Web Interface to generate a table of (x, y, z) coordinates for all planets for a series of time steps. This can then be used to validate the Keplerian calculation code.
- Fundamental Catalogues / Academic Sources:
    - For the analytical (Keplerian) method, use the full list of mean orbital elements (and their secular variation rates) for a specific epoch (e.g., J2000.0). These can often be found in astronomy textbooks or on reputable academic sites. Check JPL site (Planetary Satellites). For the mean orbital elements, click "Orbits & ephemerides" → "Mean orbital elements".

2. High-Resolution Texture Maps

These maps are used to wrap around the spherical models in Unity. Look for Equirectangular Projection maps.

- Solar System Scope Textures:
    - Source: Provides high-resolution textures based on NASA data, often optimized for 3D use, including diffuse, normal, and specular maps.

3. Rotational Data

You need the sidereal rotation period and the axial tilt (obliquity) for each planet. This information can be accessed here. The report available from this link is the report made by a working group of IAU (International Astronomical Union (IAU) Working Group on Cartographic Coordinates and Rotational Elements). This is the authoritative, formal source for planetary cartographic standards, including the official values for rotational elements and their variation over time. Planetary obliquities are not constant, therefore, these values expressed in the report have time-dependent components. For the simulation purposes, we can ignore the time-dependent parts. In essence, this report provides the direction of each planet's rotation axis (α_0, δ_0) in the celestial sphere.